Reflections in Bounded Symmetric Domains (Abstract of PhD Thesis)

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This PhD thesis is the missing completion of [1]. In [1] E. Gottschling described the role of reflections of complex manifolds in detail, and coped with the task of determining all reflections of bounded symmetric domains except for the two irreducible exceptional domains of dimensions 16 and 27. The result of this thesis is:

> Both irreducible exceptional bounded symmetric domains admit no reflections.

For the 16-dimensional domain R_{16} we use the canonical realization by M. Ise [4, 5]. However we don't need the explicit form of the domain as given in [5], but only the theorem of Ise/Yokonuma on the automorphisms of bounded symmetric domains in their canonical realization [4, p. 123/124]. From this we conclude:

> The stabilizer of the point $0 \in R_{16} \subseteq \mathbb{C}^{16}$ exactly consists of the linear mappings
>
> $$z \mapsto \lambda \cdot \rho(s)z \quad \text{for } z \in \mathbb{C}^{16},$$
>
> where $\lambda \in \mathbb{C}$, $|\lambda| = 1$, $s \in \mathrm{Spin}(10, \mathbb{R})$, and ρ is one of the two halfspin representations of $\mathrm{Spin}(10, \mathbb{R})$.

Applying Chevalleys classical theory of spinors to this intermediate result we get the nonexistence of reflections in R_{16}.

The 27-dimensional exceptional domain R_{27} is of conical type (i. e. analytically isomorphic with a Siegel halfspace of the first kind). The domains of this kind together with their automorphisms have a very concise description by formally real Jordan algebras following U. Hirzebruch[2, 3]. Using this description we develop a self-contained theory for the reflections of these domains that comprises a part of Gottschlings results independently from [1]. The main tools of this approach are the idempotents of the Jordan algebras in question: We explore the effect of reflections on idempotents. Let us denote, as usual, the maximum length r of a complete orthogonal system of idempotents of a formally real Jordan algebra as the degree of this algebra. Then the result is:

- If $r = 1$, then the associated domain has reflections of any order.

- If $r = 2$, then the associated domain has exactly one conjugacy class of reflections; these have order 2.
- If $r \geq 3$, then the associated domain admits no reflections.

The domain R_{27} belongs to the one exceptional formally real JORDAN algebra. Because this algebra has degree 3, the domain is free from reflections.

References

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[5] M. Ise; Realization of irreducible bounded symmetric domain of type (V); Proc. Japan. Acad. 45 (1969), 233–237.